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On "The generalised plane strain deformations of thick anisotropic composite laminated plates"

We noticed with great interest the recent paper by Vel and Batra (2000) which, based on the Eshelby–Stroh formalism, presented an analytical elasticity solution, as well as corresponding numerical results, for the plane strain problem of a clamped–clamped anisotropic laminated plate subjected to a certain kind of sinusoidal lateral loading. This new development comes as a nice addition to the well-known and considerably simpler Pagano's (1969) elasticity solution, which however deals with corresponding plates having both their edges simply supported.

Our interest arose from the fact that, as also cited in Vel and Batra (2000), we are involved in the development of a relevant, new but approximate stress analysis method (Soldatos and Watson, 1997a,b). This can produce corresponding but, of course, approximate numerical results on the basis, however, of a simpler computer code. Moreover, our interest was particularly enhanced from the fact that, as is described in what follows, we are currently working to further improve our new method's accuracy. Hence, the appearance of Vel and Batra (2000) gives us the opportunity to test the performance of our approach.

Our stress analysis method (Soldatos and Watson, 1997a,b) is based on a generalised, approximate plate theory that takes both transverse shear and transverse normal deformation into consideration by means of certain 'shape functions' of the transverse co-ordinate parameter. In the case of the plane strain problem considered by Vel and Batra (2000), these shape functions can incorporate entirely Pagano's (1969) displacement solution into that generalised plate theory. Hence, as far as simply supported plates are concerned, the plate theory yields displacement and stress distributions, which are identical to those of Pagano (1969). On the basis of Saint Venant's principle, it is then reasonable for someone to expect that, when dealing with other sets of boundary conditions, very accurate stress distributions can be obtained away from the edges, by solving one-dimensional (ordinary) differential equations only. Under these considerations, quite extensive numerical results were presented in Soldatos and Watson (1997a) for clamped–clamped plates, and in Soldatos and Watson (1997b) for plates having one edge clamped and the other free or guided.

The manner in which we are currently attempting to further improve the accuracy of our approximate method has already been employed by other investigators (Reddy, 1984; DiSciuva, 1986; Noor et al., 1989, 1999; Savithri and Varadan, 1990; Heuer, 1992), mostly for the improvement of the accuracy of transverse shear stress distributions predicted through certain, conventional shear deformable plate theories. In more detail, working on a predictor-corrector basis, the initially predicted in-plane stresses are substituted into the differential equations of three-dimensional elasticity which, in the corrector phase, are integrated through the plate thickness. The resulting transverse shear and transverse normal stress distributions are expected to be improvements and, therefore, more accurate predictions of their initially predicted counterparts. This, however, can essentially be verified only through appropriate comparisons with corresponding results based on alternative analytical elasticity solutions, like the results presented by Vel and Batra (2000) for clamped–clamped plates.

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Table 1
Comparison of corresponding numerical results for increasing values of length-to-thickness ratio

L/h	$E_t W(L/2,h/2)/Lq_0$		$\sigma_{11}(L/2,h)/q_0$		$\sigma_{13}(L/4,h/2)/q_0$			$\sigma_{33}(L/2,h/2)/q_0$			$E_t W(L/2,h)/Lq_0 - E_t W(L/2,0)/Lq_0$	
	Vel and Batra (2000)	Present	Vel and Batra (2000)	Present	Vel and Batra (2000)	Present		Vel and Batra (2000)	Present		Vel and Batra (2000)	Present predictor
						Predictor	Corrector		Predictor	Corrector		
4	−0.9565	−0.8605	−7.8192	−8.4211	−1.1060	−1.2142	−1.1781	−0.490	−0.513	−0.493	−0.1156	−0.1206
6	−1.6010	−1.4967	−12.7152	−12.9456	−1.8066	−1.9242	−1.9164	−0.497	−0.531	−0.498	−0.0777	−0.0829
8	−2.4003	−2.2922	−19.1168	−19.1296	−2.5280	−2.6216	−2.6200	−0.499	−0.515	−0.499	−0.0584	−0.0603
10	−3.4020	−3.2930	−27.1600	−27.0400	−3.2460	−3.3110	−3.3110	−0.500	−0.431	−0.500	−0.0467	−0.0403
15	−7.1246	−7.0133	−54.8325	−54.6750	−4.9995	−5.0190	−5.0190	−0.500	−0.185	−0.500	−0.0312	−0.0116
20	−13.2160	−13.1040	−93.5200	−93.4000	−6.7120	−6.7180	−6.7180	−0.500	−0.148	−0.500	−0.0234	−0.0069
30	−35.6940	−35.5860	−203.9400	−203.9400	−10.1040	−10.1070	−10.1070	−0.500	−0.159	−0.500	−0.0156	−0.0050
40	−77.1200	−77.0560	−358.5600	−358.5600	−13.4880	−13.4880	−13.4880	−0.500	−0.168	−0.500	−0.0117	−0.0039
60	−242.3520	−242.1360	−800.2800	−800.2800	−20.2440	−20.2440	−20.2440	−0.500	−0.175	−0.500	−0.0078	−0.0027

With the purpose to test the reliability of our approach, we are comparing in Table 1 our numerical results, obtained by means of both the predictor and the corrector phase, with the corresponding numerical results tabulated in Vel and Batra (2000). The comparisons shown in Table 1 are for homogeneous orthotropic plates having the following material properties:

$$E_L/E_T = 25, \quad G_{LT}/E_T = 0.5, \quad G_{TT}/E_T = 0.2, \quad v_{LT} = v_{TT} = 0.25. \tag{1}$$

It should be noted that these material properties differ considerably from the ones used in Soldatos and Watson (1997a,b) and, therefore, the numerical results denoted in Table 1 as 'present results' are new. Moreover, all the results presented in Table 1 are tabulated by means of the non-dimensional quantities employed in Soldatos and Watson (1997a,b). These have been found preferable to those employed in Vel and Batra (2000), because they show in a clearer manner, the difference, in order(s) of magnitude, between the bending stress, σ_{11}, and the transverse stresses, σ_{13} or σ_{33}.

As far as transverse displacements and bending stresses are concerned, the numerical results provided by our approach can be evidently based on its predictor phase only. As compared with corresponding results due to Vel and Batra (2000), there appears to be a good to excellent agreement of corresponding displacement and bending stress values, particularly for $L/h > 6$. Since two-dimensional plate theories are generally known to be inadequate for the accurate prediction of through thickness displacement and stress distributions, while their applicability in other types of problems (e.g. vibrations, buckling) is not unconditionally trusted for $L/h < 6.5$ (approximate span limit of moderately thick plates), we regard this agreement between the corresponding transverse displacement and particularly bending stress results as a very successful output of our approximate stress analysis method.

This conclusion is further verified by the comparisons shown in Table 1 between corresponding transverse shear stress predictions. The fact that for relatively thin plates ($L/h > 8$) the corrector phase cannot improve any further the value of the initial prediction is a consequence of the evident fact that, for thin plates, the initially predicted stress values are already very accurate. Apart from the case of the thickest plate considered ($L/h = 4$), the differences between the present and the exact approach predictions are within acceptable engineering limits. Moreover, the trend of the corrector phase is to move the initially predicted transverse shear stress values towards their exact elasticity counterparts (Vel and Batra, 2000).

In contrast to the outlined observations, both the initially predicted value of the transverse normal stress, σ_{33}, and the difference of the transverse displacement from the top to the bottom lateral plane, appear always to be relatively inaccurate, with the inaccuracy surprisingly increasing with decreasing plate thicknesses. It should be noted however, that although the exact value of σ_{33} remains essentially constant with decreasing plate thicknesses, the corresponding values of W, σ_{11} and σ_{13} increase remarkably. Hence, with σ_{33} being essentially two to four orders of magnitude smaller than either W or σ_{11}, small errors in the prediction of the later quantities are substantially magnified during the prediction of the quantities measured in the last two columns of Table 1. Under these considerations, it is of particular importance to further notice that this inaccuracy detected in the initial prediction of σ_{33} has essentially disappeared in the corrector phase, which yields practically the exact transverse normal stress value, regardless of the value of the aspect ratio L/h. Moreover, with W remaining practically constant throughout the thickness of a thin plate, a slight inaccuracy in measuring the difference that values of W take on the lateral plate planes is not a considerable disadvantage of the approximate stress analysis method developed in Soldatos and Watson (1997a,b).

It is finally noted that, apart from the plate cross-sections which are in the vicinity of the plate edges, all the stress distributions plotted in Vel and Batra (2000) look very similar to those obtained on the basis of our analysis. Solving however only one-dimensional (ordinary) differential equations, the order of which differs from the order of the partial differential equations solved by Vel and Batra (2000), our analysis treats the edge boundary conditions in a different, through thickness averaged manner. As a result, we never expected that our approach could predict through thickness accurate stress distributions near the plate edges. Instead, we should feel more than satisfied if, in accordance with Saint Venant's principle, the stress distributions that our approach can predict at a plate edge are, approximately, statically equivalent to those obtained by means of the exact elasticity solution (Vel and Batra, 2000).

In closing, we would like to congratulate the authors Vel and Batra (2000), for the nice and important piece of work that they added to the literature. This clearly shows the enormous benefit the traditional methods of mathematical analysis can bring into the field when they are combined with the continuously increasing power of modern computers. It also shows that, through appropriate numerical comparisons, corresponding approximate and/or numerical methods, which are eventually more advanced than their analytical counterparts, can also receive this benefit and can be assisted to develop further.

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